DFTT 13/94
February 1994

A new method for computing helicity amplitudes

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Abstract

An helicity formalism for perturbative calculations is presented. It is based on the formal insertion in spinor lines of a complete set of states built up with unphysical spinors. It is particularly convenient when massive spinors are present. Its application to $e^+ e^- \to b\,\bar{b}\,W^+W^-$ is briefly discussed.

To appear in Proceedings Nonlinear Phenomena in Complex Systems.
Polatsk Belarus. February 94

Introduction

In high energy collisions many particles or partons widely separated in phase space are often produced. The calculation of cross sections for these processes is made difficult by the large number of Feynman diagrams which appear in the perturbative expansion. This is due both to the complexity of non abelian theories and to simple combinatorics, which generates more and more diagrams when the number n of the external particles grows. Take for example a simple graph in which n external vector particles are attached to a fermion line. Exchanging of the attachments results in $n!$ graphs which contribute to the process.

If one computes unpolarized cross sections for a process with a great number (N) of Feynman diagrams with the textbook method of considering the amplitude modulus squared $|A|^2$ and taking the traces, one might end up with a prohibitive number of traces to compute. In the simple case in which there is only one fermion line per diagram, one trace has to be evaluated for every term of $|A|^2$, which amounts to $N(N+1)/2$ traces. Even if often some of them are equal, or easy to compute, this can be a very big number.

The calculation of these processes becomes simpler if one uses the so called helicity-amplitudes techniques. In such approach, given an assigned helicity to external particles, one computes the contribution of every single diagram k as a complex number a_k, sums over all k's and takes the modulus squared. To obtain unpolarized cross sections one simply sums the modulus squared for the various external helicities.

The use of helicity amplitude techniques in high energy physics dates back to the works of ref. [1]. It has been developed by the Calkul Collaboration [2] and by other authors [3-8]. An improved version of the formalism of the Calkul collaboration has been presented in [9, 10].

We describe in the following a method [11] which is in our opinion relatively fast and easy expecially when massive particles are present.

Let us consider (Fig. 1) the spinor part of a massive (or massless) fermion line with n insertions.

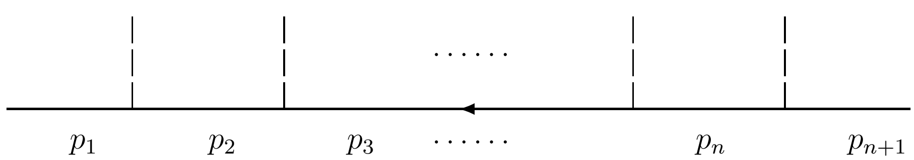

Fig. 1

It has a generic expression of the following type

$$T^{(n)} = \overline{U}(p_1, \lambda_1)\chi_1(\not{p}_2 + \mu_2)\chi_2(\not{p}_3 + \mu_3)\cdots(\not{p}_n + \mu_n)\chi_n U(p_{n+1}, \lambda_{n+1}) \qquad (1)$$

where λ_1 and λ_{n+1} are the polarizations of the external fermions, p_1 and p_{n+1} their momenta. $p_2, \ldots, p_n$ and $\mu_2, \ldots, \mu_n$ are the 4-momenta and masses appearing in the fermion propagators. $U(p, \lambda)$ ($\overline{U}(p, \lambda)$) stands for either $u(p, \lambda)$ ($\bar{u}(p, \lambda)$) or $v(p, \lambda)$

$(\bar{v}(p,\lambda))$. The χ's are

$$\chi_i^S \equiv \chi^S(c_{r_i}, c_{l_i}) = c_{r_i}\left(\frac{1+\gamma_5}{2}\right) + c_{l_i}\left(\frac{1-\gamma_5}{2}\right) \tag{2}$$

when the insertion corresponds to a scalar (or pseudoscalar), or

$$\chi_i^V \equiv \chi^V(\eta_i, c_{r_i}, c_{l_i}) = \not\eta_i\left[c_{r_i}\left(\frac{1+\gamma_5}{2}\right) + c_{l_i}\left(\frac{1-\gamma_5}{2}\right)\right] \tag{3}$$

when it corresponds to a vector particle whose 'polarization' is η. Of course η can be the polarization vector of the external particle or the vector resulting from a complete subdiagram which is connected in the i–th position to the fermion line.

There are normally three possible ways of evaluating a spinor line like this. The first consists in reducing the expression (1) to a trace and then perform its evaluation [3, 5]. This reduction is done expressing the matrix $U_\alpha(p_{n+1}, \lambda_{n+1})\,\overline{U}_\beta(p_1, \lambda_1)$ as a combination of γ matrices in a given representation. The second amounts to writing explicitely, for example in the helicity representation [6], the components of the spinors, of the $\not\eta_i$'s and of the $\not p_i$ and then proceed to the multiplication of the matrices and spinors. The other way [2, 9] consists in decomposing every $\not p_i$ in sums of external momenta $\not k_i$ and use the relation $\not k = \sum_\lambda U(k,\lambda)\overline{U}(k,\lambda) + M$ (with $M = +m$ if $U = u$, $M = -m$ if $U = v$) in order to reduce everything to the computation of expressions of the type $\overline{U}(k_i, \lambda_i)\chi U(k_j, \lambda_j)$.

We get a remarkable simplification with respect to the procedures sketched above inserting in eq.(1), just before every $(\not p_i + \mu_i)$, completeness relations formed with eigenvectors of $\not p_i$. To do this we must construct spinors $U(p,\lambda)$ which are defined also for p spacelike. With this method, in addition to reducing ourselves to the computation of expressions of the type $\overline{U}(p_i, \lambda_i)\chi U(p_j, \lambda_j)$, we avoid the proliferation of terms due to the decomposition of the $\not p_i$ in terms of external momenta.

T functions

One can easily costruct an example of spinors defined for any value of p^2 and satisfying Dirac equation and completeness relation, with a straightforward generalization of those introduced in ref.[9]. One first defines spinors $w(k_0, \lambda)$ for an auxiliary massless vector k_0 satisfying

$$w(k_0,\lambda)\bar{w}(k_0,\lambda) = \frac{1+\lambda\gamma_5}{2}\not k_0 \tag{4}$$

and with their relative phase fixed by

$$w(k_0,\lambda) = \lambda \not k_1 w(k_0, -\lambda), \tag{5}$$

with k_1 a second auxiliary vector such that $k_1^2 = -1$, $k_0 \cdot k_1 = 0$. Spinors for a four momentum p, with $m^2 = p^2$ are then obtained as:

$$u(p,\lambda) = \frac{\not p + m}{\sqrt{2\,p\cdot k_0}}\, w(k_0, -\lambda) \qquad v(p,\lambda) = \frac{\not p - m}{\sqrt{2\,p\cdot k_0}}\, w(k_0, -\lambda) \tag{6}$$

$$\bar{u}(p,\lambda) = \bar{w}(k_0,-\lambda)\,\frac{\not{p}+m}{\sqrt{2\,p\cdot k_0}} \qquad \bar{v}(p,\lambda) = \bar{w}(k_0,-\lambda)\,\frac{\not{p}-m}{\sqrt{2\,p\cdot k_0}} \tag{7}$$

If p is spacelike, one of the two determination of $\sqrt{p^2}$ has to be chosen for m in the above formulae, but physical results will not depend on this choice.

One can readily check that with the previous definitions, Dirac equations

$$\not{p}u(p) = +mu(p) \qquad \not{p}v(p) = -mv(p) \tag{8}$$

$$\bar{u}(p)\not{p} = +m\bar{u}(p) \qquad \bar{v}(p)\not{p} = -m\bar{v}(p) \tag{9}$$

and completeness relation

$$1 = \sum_{\lambda} \frac{u(p,\lambda)\bar{u}(p,\lambda) - v(p,\lambda)\bar{v}(p,\lambda)}{2m} \tag{10}$$

are satisfied also when $p^2 \leq 0$ and m is imaginary.

Let us now consider the case in which there are only two insertions in a spinor line:

$$T^{(2)}(p_1;\chi_1;p_2;\chi_2;p_3) = \overline{U}(p_1,\lambda_1)\chi_1(\not{p}_2+\mu_2)\chi_2 U(p_3,\lambda_3). \tag{11}$$

One can insert in eq. (11), on the left of $(\not{p}_2+\mu_2)$ the relation (10) and make use of Dirac equations to get:

$$\begin{aligned} T^{(2)} \;=\; & \frac{1}{2}\overline{U}(p_1,\lambda_1)\chi_1 u(p_2,\lambda_2)\times\bar{u}(p_2,\lambda_2)\chi_2 U(p_3,\lambda_3)\times\left(1+\frac{\mu_2}{m_2}\right)+ \\ & \frac{1}{2}\overline{U}(p_1,\lambda_1)\chi_1 v(p_2,\lambda_2)\times\bar{v}(p_2,\lambda_2)\chi_2 U(p_3,\lambda_3)\times\left(1-\frac{\mu_2}{m_2}\right) \end{aligned} \tag{12}$$

This example can be generalized to any number of insertions and shows that the factors $(\not{p}_i+\mu_i)$ can be eliminated, reducing all fermion lines essentially to products of T functions:

$$T_{\lambda_1\lambda_2}(p_1;\chi;p_2) = \overline{U}(p_1,\lambda_1)\chi U(p_2,\lambda_2) \tag{13}$$

defined for any value of p_1^2 and p_2^2.

The T functions (13) have a simple dependence on m_1 and m_2 and as a consequence the rules for constructing spinor lines out of them are simple. They can in fact, using eqs.(6,7) be written as:

$$\begin{aligned} \widetilde{T}_{\lambda_1\lambda_2}(p_1;\chi;p_2) \equiv \sqrt{p_1\cdot k_0}\,\sqrt{p_2\cdot k_0}\;T_{\lambda_1\lambda_2}(p_1;\chi;p_2) = \\ A_{\lambda_1\lambda_2}(p_1;\chi;p_2) + M_1 B_{\lambda_1\lambda_2}(p_1;\chi;p_2) + M_2 C_{\lambda_1\lambda_2}(p_1;\chi;p_2) + M_1 M_2 D_{\lambda_1\lambda_2}(p_1;\chi;p_2) \end{aligned} \tag{14}$$

where

$$\begin{aligned} M_i = +m_i \qquad & \text{if} \qquad U(p_i,\lambda_i) = u(p_i,\lambda_i) \\ M_i = -m_i \qquad & \text{if} \qquad U(p_i,\lambda_i) = v(p_i,\lambda_i). \end{aligned} \tag{15}$$

The functions A, B, C, D turn out to be independent of m_1 and m_2 and of the u or v nature of $\overline{U}(p_1,\lambda_1)$ and $U(p_2,\lambda_2)$. We give in Appendix A the expressions for A^V, B^V, C^V, D^V and A^S, B^S, C^S, D^S, which are the A, B, C, D functions for a vector and a scalar insertion respectively .

From T functions to spinor lines

The functions

$$\widetilde{T}^{(n)} = T^{(n)} \sqrt{p_1 \cdot k_0}\, \sqrt{p_{n+1} \cdot k_0}\, (p_2 \cdot k_0)\, (p_3 \cdot k_0) \cdots\, (p_n \cdot k_0). \tag{16}$$

can be computed recursively starting from T functions (or $\widetilde{T} = \widetilde{T}^{(1)}$). The $T^{(n)}$ themselves, and hence the complete spinor line, can then be immediately obtained at the end of the computation, dividing by the appropriate factors.

Let us denote with $\widetilde{T}, A, B, C, D$ the $2x2$ matrices whose elements are $\widetilde{T}_{\lambda_1\lambda_2}$, $A_{\lambda_1\lambda_2}$, $B_{\lambda_1\lambda_2}$, $C_{\lambda_1\lambda_2}$, $D_{\lambda_1\lambda_2}$. With this notation, making use of eqs. (14) and (13), eq. (12) reads:

$$\begin{aligned}
\widetilde{T}^{(2)}(1,2,3) = \frac{1}{2}\Big[&\Big(A(1,2) + M_1 B(1,2) + m_2 C(1,2) + M_1 m_2 D(1,2)\Big) \\
&\times \left(1 + \frac{\mu_2}{m_2}\right) \times \Big(A(2,3) + m_2 B(2,3) + M_3 C(2,3) + m_2 M_3 D(2,3)\Big) + \\
&\Big(A(1,2) + M_1 B(1,2) - m_2 C(1,2) - M_1 m_2 D(1,2)\Big) \times \\
&\left(1 - \frac{\mu_2}{m_2}\right) \times \Big(A(2,3) - m_2 B(2,3) + M_3 C(2,3) - m_2 M_3 D(2,3)\Big)\Big]
\end{aligned} \tag{17}$$

where we have used the shorthands $(1,2)$ and $(1,2,3)$ for $(p_1;\chi_1;p_2)$ and $(p_1;\chi_1;p_2;\chi_2;p_3)$ respectively.

It is immediate to see that $\widetilde{T}^{(2)}$ has again the same dependence on the external masses as in (14):

$$\widetilde{T}^{(2)}(1,2,3) = A^{(2)}(1,2,3) + M_1 B^{(2)}(1,2,3) + M_3 C^{(2)}(1,2,3) + M_1 M_3 D^{(2)}(1,2,3) \tag{18}$$

with

$$\begin{aligned}
A^{(2)}(1,2,3) &= A(1,2)\Big(A(2,3) + \mu_2 B(2,3)\Big) + C(1,2)\Big(\mu_2 A(2,3) + p_2^2 B(2,3)\Big) \\
B^{(2)}(1,2,3) &= B(1,2)\Big(A(2,3) + \mu_2 B(2,3)\Big) + D(1,2)\Big(\mu_2 A(2,3) + p_2^2 B(2,3)\Big) \\
C^{(2)}(1,2,3) &= A(1,2)\Big(C(2,3) + \mu_2 D(2,3)\Big) + C(1,2)\Big(\mu_2 C(2,3) + p_2^2 D(2,3)\Big) \\
D^{(2)}(1,2,3) &= B(1,2)\Big(C(2,3) + \mu_2 D(2,3)\Big) + D(1,2)\Big(\mu_2 C(2,3) + p_2^2 D(2,3)\Big)
\end{aligned} \tag{19}$$

This implies that $A^{(2)}$, $B^{(2)}$, $C^{(2)}$, $D^{(2)}$ can be reinserted in an equation like eq. (17) to give the $\widetilde{T}$ function $\widetilde{T}^{(3)}$ corresponding to a fermion line with 3 insertions, and so on. So one can generalize eqs. (17,18,19) by induction. Every $\widetilde{T}^{(i)}$ will turn out to be of the form

$$\widetilde{T}^{(i)} = A^{(i)} + M_1 B^{(i)} + M_{i+1} C^{(i)} + M_1 M_{i+1} D^{(i)}. \tag{20}$$

From this it follows that the evaluation of any spinor line can be performed computing the A, B, C, D matrices relative to every single insertion and combining them toghether until one gets to the final $T^{(n)}$. The formalism can be conveniently cast in matrix

notation, since every piece of a spinor line as well as every complete spinor line with n insertions is completely known when we know the matrix

$$\tau = \begin{pmatrix} A & C \\ B & D \end{pmatrix} \tag{21}$$

The law of composition of two pieces of spinor line , connected by a fermion propagator with 4-momentum p and mass μ, whose matrices are

$$\tau_1 = \begin{pmatrix} A_1 & C_1 \\ B_1 & D_1 \end{pmatrix} \qquad\qquad \tau_2 = \begin{pmatrix} A_2 & C_2 \\ B_2 & D_2 \end{pmatrix},$$

is simply (cfr. eq. (19)) :

$$\begin{pmatrix} A & C \\ B & D \end{pmatrix} = \begin{pmatrix} A_1 & C_1 \\ B_1 & D_1 \end{pmatrix} \begin{pmatrix} 1 & \mu \\ \mu & p^2 \end{pmatrix} \begin{pmatrix} A_2 & C_2 \\ B_2 & D_2 \end{pmatrix}. \tag{22}$$

If we call π_i the matrix

$$\pi_i = \begin{pmatrix} 1 & \mu_i \\ \mu_i & p_i^2 \end{pmatrix} \tag{23}$$

corresponding to the propagator of 4-momentum p_i and τ_i the matrix associated with the i-th insertion of fig. 1, the τ matrix of the whole spinor line can then be computed as follows:

$$\tau = \tau_1 \pi_2 \tau_2 \pi_3 \tau_3 \cdots \pi_{n-1} \tau_{n-1} \pi_n \tau_n \tag{24}$$

When one has to deal with massless spinor lines, all the formulae given in the appendix A remain valid. But in this case the fact that all μ_i's as well as m_1 and m_{n+1} are zero leads to significant simplifications. The π_i matrices (23) become diagonal and moreover it is not necessary to know the whole $\tau^{(n)}$ matrix to compute the spinor line with n insertions. Only $A^{(n)}$ is needed.

An application: $e^+ e^- \rightarrow b \bar{b} W^+ W^-$

We have tested the method for helicity amplitude computations just described and we have always found perfect numerical agreement with the other methods. We think it has some advantages over the ones mentioned in the introduction and it surely turned out to be faster in our tests. Recently we have used it for computing $e^+ e^- \rightarrow b \bar{b} W^+ W^-$[12]. Some representative tree level diagrams for this process are reported in Fig. 2. From them one can see that the W's can be both attached to the massive fermion line directly or through a trilinear coupling with γ,Z or Higgs. They can also be both attached to the massless fermion line, or one to the massless and one to the massive line. Finally one or both of them can be attached through a trilinear or quadrilinear vector coupling or through a Higgs to the intermediate vector propagators. Altoghether the process is described by 61 diagrams. We have performed tests of gauge invariance on the amplitudes and we have also computed the same amplitudes with the method of ref. [6] as a check of our results. We found out in this case that the computation with our method was about four times faster.

The phenomenological interest of the reaction above lies principally in the studies of top and Higgs production at the Next Linear Collider. This can be easily understood from the first and last diagram of fig. 2. In the first diagram a $t\bar{t}$ pair is produced with the subsequent decay of t to W^+b and $\bar{t}$ to $W^-\bar{b}$. A top heavier than 100 GeV is expected to decay to $W\,b$ before any hadronization takes place and hence, with the present limits on the top mass, the final state under scrutiny is precisely what one expects to detect in $t\,\bar{t}$ production. Similarly the last diagram corresponds to $e^+\,e^- \to H\,Z$ with subsequent decay of the Higgs in two W's and of the Z in $b\,\bar{b}$. The production of H in association with Z is the most favourable mechanism for Higgs production at an $e^+\,e^-$ collider with an energy $\leq$ 500 GeV and the decay channel $H \to WW$ is the one with the largest branching ratio for a Higgs mass above the WW threshold. From the point of view of $t\,\bar{t}$ (Higgs) production all diagrams except the first (last) in fig. 2 have to be considered as background. Our amplitudes allow a complete and coherent study of signal and background for these processes, taking into account interference and finite width effects. They also allow the study of possible interplays between the two processes which can be significant for particular values of the top and Higgs masses . Our results concerning these processes can be found in ref. [12].

Conclusions

We have described a new method for helicity calculations based on the insertion in spinor lines of completeness relations formed with generalized spinors. The technique has been tested with success and it appears to be particularly fast when massive spinors are involved. An application to a reaction relevant for top and Higgs measurements has been reported.

Appendix A

Using the spinors (6,7, 15), their products can be written as

$$
\begin{aligned}
U(p_2,\lambda_2)\overline{U}(p_1,\lambda_1) &= \frac{1}{4\sqrt{2\,p_1\cdot k_0}\sqrt{2\,p_2\cdot k_0}} \\
&\times(\not{p}_2 + M_2)\left[(1+\lambda_1\lambda_2) - (\lambda_1+\lambda_2)\gamma^5 + \not{k}_1[(\lambda_1-\lambda_2) - (1-\lambda_1\lambda_2)\gamma^5]\right]\not{k}_0(\not{p}_1 + M_1).
\end{aligned}
\tag{A.1}
$$

Multiplying to the right by χ^S or χ^V of eqs. (2,3) , taking the trace and with the help of eq. (14), one gets the following expressions for the functions A, B, C, D corresponding to a scalar and a vector insertion :

$$
\begin{aligned}
A^S_{+-} &= c_l\,(k_0\cdot p_1\; k_1\cdot p_2 - k_0\cdot p_2\; k_1\cdot p_1 - i\epsilon\,(k_0,k_1,p_1,p_2)) \\
A^S_{-+} &= c_r\,(-k_0\cdot p_1\; k_1\cdot p_2 + k_0\cdot p_2\; k_1\cdot p_1 - i\epsilon\,(k_0,k_1,p_1,p_2)) \\
B^S_{++} &= c_r\; k_0\cdot p_2 \qquad\qquad B^S_{--} = c_l\; k_0\cdot p_2 \\
C^S_{++} &= c_l\; k_0\cdot p_1 \qquad\qquad C^S_{--} = c_r\; k_0\cdot p_1 \\
A^V_{++} &= c_r\,(-k_0\cdot\eta\; p_1\cdot p_2 + k_0\cdot p_1\; \eta\cdot p_2 + k_0\cdot p_2\; \eta\cdot p_1 + i\epsilon\,(k_0,\eta,p_1,p_2))
\end{aligned}
$$

$$\begin{aligned}
A^V_{--} &= c_l\,(-k_0\cdot\eta\; p_1\cdot p_2 + k_0\cdot p_1\; \eta\cdot p_2 + k_0\cdot p_2\; \eta\cdot p_1 - i\epsilon\,(k_0,\eta,p_1,p_2)) \qquad \text{(A.2)}\\
B^V_{+-} &= c_l\,(k_0\cdot\eta\; k_1\cdot p_2 - k_0\cdot p_2\; k_1\cdot\eta - i\epsilon\,(k_0,k_1,\eta,p_2))\\
B^V_{-+} &= c_r\,(-k_0\cdot\eta\; k_1\cdot p_2 + k_0\cdot p_2\; k_1\cdot\eta - i\epsilon\,(k_0,k_1,\eta,p_2))\\
C^V_{+-} &= c_r\,(-k_0\cdot\eta\; k_1\cdot p_1 + k_0\cdot p_1\; k_1\cdot\eta + i\epsilon\,(k_0,k_1,\eta,p_1))\\
C^V_{-+} &= c_l\,(k_0\cdot\eta\; k_1\cdot p_1 - k_0\cdot p_1\; k_1\cdot\eta + i\epsilon\,(k_0,k_1,\eta,p_1))\\
D^V_{++} &= c_l\; k_0\cdot\eta \qquad\qquad D^V_{--} = c_r\; k_0\cdot\eta.
\end{aligned}$$

All functions A, B, C, D for a single insertion not reported in the preceding list are identically zero.
The function ϵ is defined to be the determinant:

$$\epsilon\,(p,q,r,s) = det \begin{vmatrix} p^0 & q^0 & r^0 & s^0 \\ p^1 & q^1 & r^1 & s^1 \\ p^2 & q^2 & r^2 & s^2 \\ p^3 & q^3 & r^3 & s^3 \end{vmatrix} \qquad \text{(A.3)}$$

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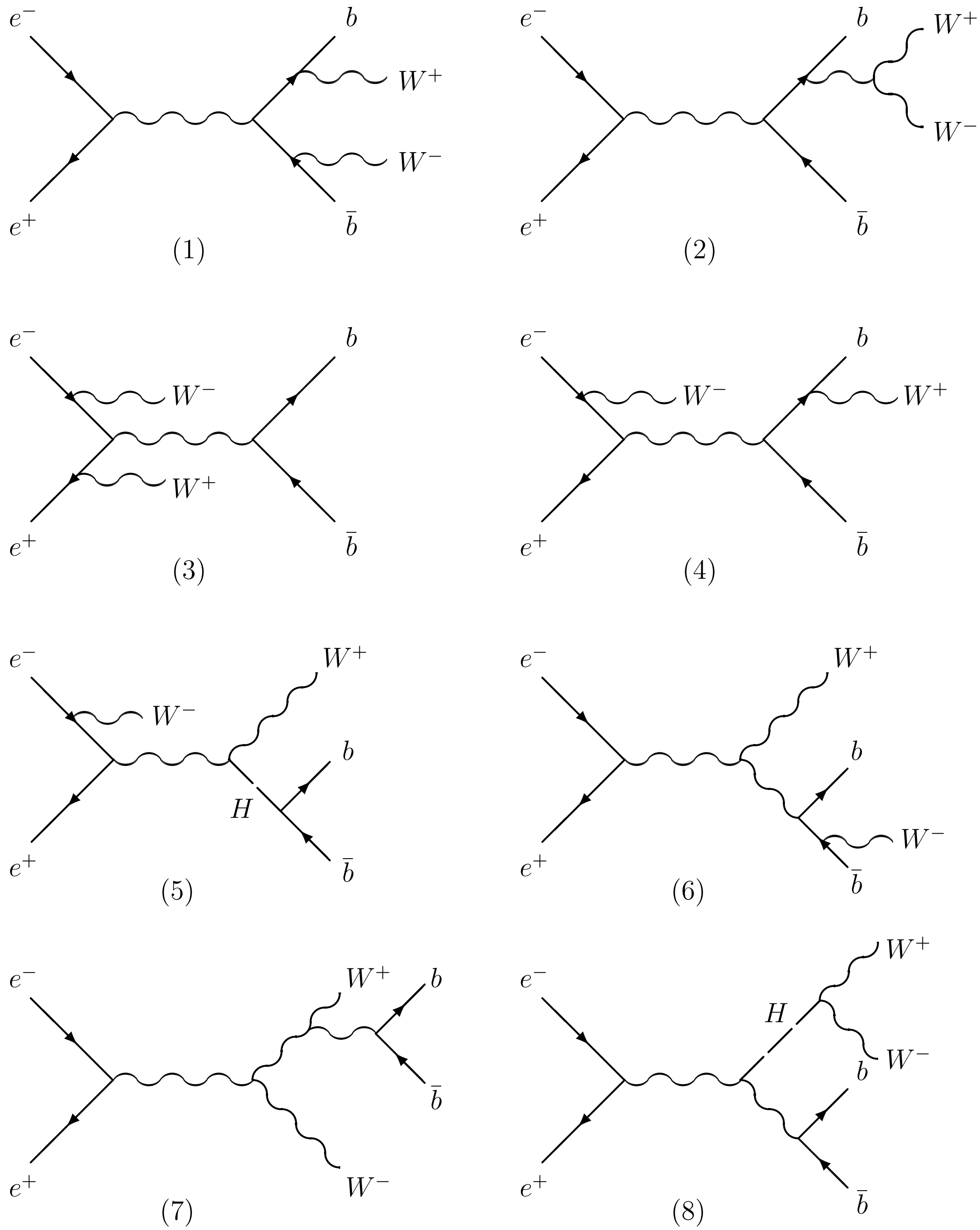
e^- b W^+ W^- e^+ $\bar{b}$
(1)
e^- b W^+ W^- e^+ $\bar{b}$
(2)
e^- W^- b W^+ e^+ $\bar{b}$
(3)
e^- W^- b W^+ e^+ $\bar{b}$
(4)
e^- W^+ W^- b H e^+ $\bar{b}$
(5)
e^- W^+ b W^- e^+ $\bar{b}$
(6)
e^- W^+ b $\bar{b}$ W^- e^+
(7)
e^- W^+ H W^- b e^+ $\bar{b}$
(8)

Fig. 2